USN ☐☐☐☐☐☐☐☐☐☐ 15CS36

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Discrete Mathematical Structures

Time: 3 hrs.   Max. Marks: 80

Note: Answer any FIVE full questions, choosing
ONE full question from each module.

Module-1

1   a.   Define Tautology. Verify the following compound proposition is a tautology or not :
         $\{(p \vee q) \to r\} \leftrightarrow \{\sim r \to \sim (p \vee q)\}$.   **(04 Marks)**

    b.   Check whether the following argument is valid or not :
         If I study, then I will not fail in exam.
         If I do not watch TV in the evenings, then I will study.
         I failed in exam.
         $\therefore$ I must have watched TV in the evenings.   **(04 Marks)**

    c.   Define : i) open sentence ii) quantifiers. Write the following proposition in symbolic form
         and find its negation : "All integers are rational numbers and some rational numbers are
         integers".   **(04 Marks)**

    d.   Give a direct proof of the statement, "For all integers K and ℓ, if K and ℓ are both even then
         $K + \ell$ is even and $K\ell$ is even".   **(04 Marks)**

OR

2   a.   Define converse, inverse and contra positive of an implication. Hence find converse, inverse
         and contra positive for "$\forall x, (x > 3) \to (x^2 > 9)$" where universal set is the set of real
         numbers R.   **(04 Marks)**

    b.   Using the laws of logic, prove the following logical equivalence :
         $[(\sim p \vee \sim q) \wedge (F_0 \vee p) \wedge P] \Leftrightarrow p \wedge \sim q$.   **(04 Marks)**

    c.   What are bound variables and free variables. Identify the same in each of the following
         expressions :
          i) $\forall y, \exists z \{\cos(x + y) = \sin(z - x)\}$
          ii) $\exists x, \exists y \{(x^2 - y^2) = z\}$.   **(04 Marks)**

    d.   Verify the validity of the following argument : If a triangle has two equal sides, then it is
         isosceles. If a triangle is isosceles, then it has two equal angles. The triangle $\triangle ABC$ does not
         have two equal angles. $\therefore \triangle ABC$ does not have two equal sides.   **(04 Marks)**

Module-2

3   a.   Prove by mathematical induction $1.3 + 2.4 + 3.5 + \cdots + n(n + 2) = \dfrac{n(n+1)(2n+7)}{6}$.

                                                                                                     (04 Marks)

    b.   Give a recursive definition for each of the following integer sequence :
         i) $a_n = 7n$ ii) $a_n = 2 - (-1)^n$ for $n \in z^+$.   **(04 Marks)**

    c.   How many positive integers can be formed by using the digits 3, 4, 4, 5, 5, 6, 7 to exceed
         5,000,000?   **(04 Marks)**

    d.   In how many ways can we distribute seven apples and six oranges among four children so
         that each child receives at least one apple?   **(04 Marks)**

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OR

4 a. If F_0, F_1, F_2, - - - - are Fibonacci numbers, then prove by induction $\displaystyle\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$.

(04 Marks)

   b. A sequence $\{a_n\}$ is defined recursively as $a_1 = 7$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$. Find a_n in explicit form. **(04 Marks)**

   c. Find the number of arrangements of all the letters in the word "TALLAHASSEE". How many of these arrangements have no adjacent A's? **(04 Marks)**

   d. Find the coefficient of $w^3 x^2 y z^2$ in the expansion of $(2w - x + 3y - 2z)^8$. **(04 Marks)**

Module-3

5 a. Define Cartesian product of two sets. For any three non-empty sets A, B and C. Prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(04 Marks)**

   b. Let f and g be two functions form R to R defined by $f(x) = 2x + 5$ and $g(x) = \dfrac{x-5}{2}$. Show that f and g are invertible to each other. **(04 Marks)**

   c. Define partition of a set. If R is a relation defined on $A = \{1, 2, 3, 4\}$ by $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$, determine the partition induced by R. **(04 Marks)**

   d. Let $A = \{a, b, c\}$, $B = P(A)$ where P(A) is the power set of A. Let R be a subset relation on A. Show that (B, R) is a POSET and draw its Hasse diagram. **(04 Marks**

OR

6 a. Let R be an equivalence relation on set A and a, b $\in$ A. Then prove the following are equivalent :
   i) $a \in [a]$
   ii) a R b iff $[a] = [b]$
   iii) if $[a] \cap [b] \neq \phi$ then $[a] = [b]$. **(04 Marks)**

   b. Prove that a function $f : A \rightarrow B$ is invertible iff it is one – one and onto. **(04 Marks)**

   c. State Pigeonhole principle. Show that if any seven numbers from 1 to 12 are chosen, then two of them will add to 13. **(04 Marks)**

   d. Show that the set of positive divisors of 36 is a POSET and draw its Hasse diagram. Hence find its i) least element ii) greatest element. **(04 Marks**

Module-4

7 a. Out of 30 students in a hostel, 15 study history, 8 study economics and 6 study geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(04 Marks)**

   b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all these derangements. **(04 Marks)**

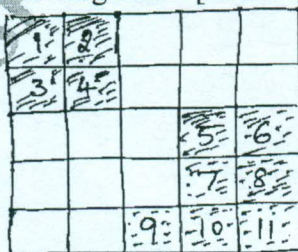   c. Find the rook polynomial for the following board [refer Fig.Q7(c)] :

Fig. Q7(c) **(04 Marks)**

   d. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(04 Marks)**

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OR

8 a. Determine the number of integers between 1 and 300 (inclusive) which are
   i) divisible by exactly two of 5, 6, 8   ii) divisible by at least two of 5, 6, 8.   (04 Marks)

 b. In how many ways can be integers 1, 2, - - - -, 10 be arranged in a line so that no even
    integer is in its natural place.   (04 Marks)

 c. An apple, a banana, a mango and an ornage are to be distributed to four boys B_1, B_2, B_3, B_4.
    The boys B_1 and B_2 do not wish to have apple, the boy B_3 does not want banana or mango,
    B_4 refuses orange. In how many ways the distribution can be made so that no boy is
    displeased?   (04 Marks)

 d. Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ given that $F_0 = 0$, $F_1 = 1$.   (04 Marks)

Module-5

9 a. Define the following with an example for each :
   i) Complete graph   ii) regular graph   iii) bipartite graph   iv) complete bipartite graph.   (04 Marks)

 b. Define isomorphism of two graphs. Verify the following graphs are isomorphic or not :
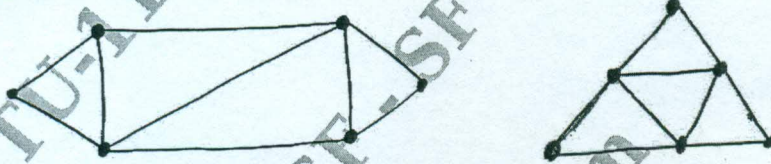   [Refer Fig.Q9(b)]   (04 Marks)

Fig.Q9(b)

 c. Show that a tree with n vertices has n – 1 edges.   (04 Marks)

 d. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies
    20, 28, 4, 17, 12, 7 respectively.   (04 Marks)

OR

10 a. Explain Konigsberg bridge problem.   (04 Marks)

 b. Define the following with an example :
    i) subgraph   ii) spanning subgraph
    iii) induced subgraph   iv) edge–disjoint and vertex – disjoint subgraphs.   (04 Marks)

 c. If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and
    one vertex of degree 5, find the number of leaves in T.   (04 Marks)

 d. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code.   (04 Marks)

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